

Final Examination, Fall – 2025

Time: 2 Hours

[The right-most column indicates the assigned marks]

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|-----|---|------|-----|
| Q1. | Explain the following with example | CO-1 | [2] |
| | (a) Analytic Function | (C2) | |
| | (b) Spectrum | | |
| | (c) Algebraic Multiplicity | (C2) | |
| | (d) Linear Transformation | | |
| Q2. | $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ is a matrix | CO-1 | [8] |
| | (C2) | | |
| | a) Identify the eigenvalues of A^T, A^3, A^{-1}, A^{-3} | | |
| | b) Identify the eigenvectors of the matrix A. | | |
| Q3. | a) Calculate whether or not the following vectors are linearly dependent | CO-2 | [5] |
| | $t^2 + 2t + 3, 3t^2 - t + 2, 5t^2 + 3t + 8$ in $\mathbb{R}^3$. | (C3) | |
| | b) Identify the following mapping is linear or not and discover T^{-1} if it exists, | CO-2 | [5] |
| | where $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is given by | (C3) | |
| | $T(x, y, z) = (x - 2y, -2z, z + x)$ | | |
| | c) Interpret whether or not the vector $(5, -3, 2, -2)$ as a linear combination of the | CO-2 | [5] |
| | vectors $(1, -1, 0, 1), (0, 2, -1, -1), (2, -1, 1, 0)$ and $(-1, 1, 2, -1)$ in $\mathbb{R}^4$. | (C3) | |
| Q4. | (a) List out all the possible roots of | CO-3 | [5] |
| | i) $z^3 = -1 + i$ | (C4) | |
| | ii) $z^4 = -2\sqrt{3} - 2i$ | | |
| | (b) Consider the complex expression $\frac{(2+i)^2}{(3-i)^2}$. First, evaluate the expression and | CO-3 | [4] |
| | represent your answer in the form $A+iB$, where A and B are real numbers. Then, | (C4) | |
| | classify the resulting complex number based on its position on the complex plane by identifying the quadrant or axis it lies on. | | |
| | (c) Using Cauchy's integral formula, analyze | CO-3 | [6] |
| | i. $\oint \frac{z}{(9-z^2)(z+i)} dz$, Where the circle is $ z = 2$ | (C4) | |
| | ii. $\oint \frac{2z-1}{z(z+2)(2z+1)} dz$, Where the circle is $ z = 1$ | | |
| | iii. $\oint \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, Where the circle is $ z = 3$ | | |

[illegible]