Daffodil International University

Faculty of Science & Information Technology
Department of Computer Science and Engineering
Final Examination, Spring-2025
Course Code: MAT 102, Course Title: Mathematics II
Level: 01 Term: 02 Batch: 67

Time: 2 Hours **Marks: 40**

Answer All Questions

[The figures in the right margin indicate the full marks and corresponding course outcomes. All portions of each question must be answered sequentially.]

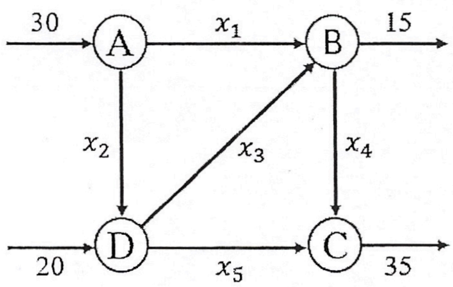
1. a) Given $A = \begin{bmatrix} 4 & 1 & 3 \\ -2 & 1 & 2 \\ 1 & 0 & 5 \end{bmatrix}$. [3+5+1]

(i) Organize A as a sum of a symmetric matrix and a skew-symmetric matrix.
(ii) Construct A^{-1} if it exists.
(iii) Identify whether the matrix A^{-1} is orthogonal or not.

b) Given the matrix $M = \begin{bmatrix} 1 & -3 & 4 & -3 & 2 & 5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{bmatrix}$, identify the Rank, construct the Row Echelon Form (REF) and the Reduced Row Echelon Form (RREF) of M. [5]

CO2

2. The figure below shows the traffic flow (vehicles per hour) through a network of streets. [2+3+1] CO3



(i) Analyze the traffic flow in the network and construct a system of linear equations that represents this network.
(ii) Examine the relationships among the variables x_1, x_2, x_3, x_4 and x_5 by solving the system of equations.
(iii) Discover the traffic flow when $x_3 = 5$ and $x_5 = 20$.

3. Given $M = \begin{bmatrix} 3 & 0 & 2 \\ 6 & 4 & 3 \\ -4 & 0 & -6 \end{bmatrix}$. [4+4] CO3

(i) List out the eigenvalues of M^{-3} and $(MM^{-1})^6$.
(ii) Inspect the trace of M^4 and the spectrum of $(M^T)^{-5}$.

4. a) Assess the linear independence of the vectors $(1, -2, 1, 3), (-2, 4, 2, 2)$ and $(3, -6, 1, 5)$. If they are dependent, find a linear dependence relation and verify it. [3+2]

b) $P(x, y, z) = (-3y, 2x + y, 3y - z)$, $Q(x, y, z) = (2x - y, xy + z, z - x)$, $R(x, y, z) = (4x, x - y, 2x - y + z)$, $S(x, y, z) = (x + y - z, 2y + z)$. [4+3]

(i) Examine which are Linear Transformations.
(ii) Evaluate RoP and PoS.

CO4